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INVENTORY MODEL (M. T.) EXPONENTIAL BACKORDER COST RANDOM SUPPLY AND CONSTANT LEAD TIME SERIES 2

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Abstract
This paper derives the inventory costs for inventory model (M.T) with exponential backorder cost and random supply and constant-lead times by taking the inventory cost for model (M.T) exponential backorder costs with constant lead times equation (12) of series 1 and averaging the inventory costs over the states of M. the maximum reorder level. The supply is assumed to be a gamma variate and demand during the lead time is still following a normal distribution.

Introduction
In series 1, exponential backorder costs and continuous lead times, the inventory cost for fixed supply was derived equation 12.
The paper makes use of this result for random supply by averaging the inventory coasts over the states of m the maximum re-order level. Supply is assumed to a gamma variate.

Literature Review
Zipkin (2006) threats both fixed and random lead times and examines both stationary and limiting distributions under different assumption.
Bartsimas (1999) in his paper probabilistic service level guarantee in make-to-stock, considered both linear and quadratic inventory costs and backorder costs.
Pritibhushan Sinha (2006) in his paper. "A note in Bernoulli demand inventory model presents a simple-item, continuous monitoring inventory model with probabilistic demand for the item and probabilistic lead time of order replacement"
Hadley and Within (1992) extensively developed the inventory model (M.T) for constant lead times and linear backorder costs.

Inventory model (M.T) exponential backorder costs, random supply and constant lead times.
The inventory cost when the maximum re-order cover in fixed is given equation series 1

$$C = \frac{Rc + S}{T} + hc\left(M - DL - \frac{DT}{2}\right) + \frac{hc}{T}(G_5(M, T, +L)) - G_5(M, L) + \frac{1}{T}(G_6(M, T + L) - G_6(M, L))$$

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Where $G_5(M,T) = \left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2} + \frac{\sigma^{2M}}{2D^2} - TM + \frac{M^2}{2D}\right) F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right)$

$+\frac{\sqrt{\sigma^{2T}}}{2}\left(\left(T - \frac{\sigma^2}{D^2}\right) - \frac{M}{D}\right) g\left(\frac{M-DT}{\sqrt{\sigma^{2T}}}\right)$

$$G_6(M,T) = \frac{2Db_1}{b_2\left(\sigma^2 b_2^2 + 2D^2 b_2\right)} \text{esp}\left[T\left(\frac{\sigma^2 b_2^2 + 2D^2 b_2}{2D^2}\right) - \frac{b_2 M}{D}\right] F\left(\frac{M - T\left(D + \frac{\sigma^2 b_2}{D}\right)}{\sqrt{\sigma^2 T}}\right)$$

$$-\frac{b_1}{b_2}\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) g\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) - \frac{\sigma^2 b_2^2 b_1}{Db_2(\sigma^2 b_2^2 + 2D^2 b_2)} \text{ esp}\left(\frac{2DM}{\sigma^2}\right) F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) - \frac{b_1}{b_2} F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right)$$

The p. df of M, $U(M) = M^{v-1}\mu^v \text{ esp}\frac{-\mu M}{\Gamma v}$ $\quad \mu > 0, \ v > 0, \ M > 0$

Multiplying $G_5(M,T)$ by u (M)

$$U(M)G_5 = \left[\left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2}\right) + \left(\frac{\sigma^2}{2D^2} - T\right) M^v + \frac{M^{v+1}}{2D}\right] \text{esp}\frac{(-\mu M)\mu^v}{\Gamma(v)} F\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right)$$

$$+\frac{\sigma T^{1/2}}{2}\left(\left(T - \frac{\sigma^2}{D^2}\right) M^{v-1} - \frac{M^v}{D}\right) g\left(\frac{M-DT}{\sqrt{\sigma^2 T}}\right) - \text{ esp}\frac{(-\mu M)\mu^v}{\Gamma(v)} M^{v-1}$$

$$\text{esp}\left(\frac{2DM}{\sigma^2}\right) F\left(\frac{M+DT}{\sqrt{\sigma^2 T}}\right)$$

Let $G_{11}(T) = \int_U^\infty U(M) G_2(M,T)\, dM$

Noting that $\int_0^\infty \frac{\exp(-MQ)Q^{v-1}\mu^v}{\Gamma(v)} F\left(\frac{R+Q-DL}{\sqrt{\sigma^2 T}}\right) dQ$

$$= \frac{\mu^V}{\Gamma(v)} \sum_{z=1}^{v} \frac{(v-1)!}{\mu^z (v-z)!} \sum_{z=0}^{v-\frac{z}{2}} \binom{v-z-i}{i} (DL - R - \mu\sigma^2 L)^{v-z-2i} \left(\frac{\sigma^2 L}{2}\right)^i$$

$$esp\left(R\mu + \frac{\mu\sigma^2 L}{2} - D\mu L\right) \tag{7}$$

Substituting for U(M) G_2 (M,T), integrating and applying

$$G_{11}(T) = \frac{\mu^v \text{esp}\,(\mu^2\sigma^2 T - D\mu T)}{\Gamma(v)} \left[\left(\frac{\sigma^4}{4D^3} + \frac{DT^2}{2}\right) \sum_{z=1}^{v} \sum_{i=0}^{\frac{v-2}{2}} \frac{(v-1)!}{\mu^z (v-2)!} \binom{v-z-i}{i}\right.$$

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$$(DT-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2}{2}\right)^i+\frac{1}{2D}\sum_{z=1}^{v+2}\frac{(v+1)!}{\mu^z(v+2-z)!}\sum_{i=0}^{\frac{v+2-z}{2}}\binom{v+1-z-i}{i}(DT-\mu\sigma^2 T)^{v+1-z-2i}$$

$$\left.\left(\frac{\sigma^2 T}{2}\right)^i\right]+\left(\frac{\sigma^2}{2D^2}-T\right)\sum_{z=1}^{v+1}\sum_{i=0}^{\frac{v+1-z}{2}}\frac{v!}{\mu^z(v+1-z)!}\binom{v+1-z-i}{i}(DT-\mu\sigma^2 T)^{v+1-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i$$

$$+\frac{\sigma^2 T}{2}\frac{\mu^v esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)}{\lceil(v)}\left(T-\frac{\sigma^2}{D^2}\right)\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}(DT-\mu\sigma^2 T)^{v-1-2i}\left(\frac{\sigma^2 T}{2}\right)^i-\frac{1}{D}$$

$$\sum_{i=0}^{\frac{v}{2}}\binom{v-i}{i}(DT-\mu\sigma^2 T)^{v-2i}\left(\frac{\sigma^2 T}{2}\right)^i-\frac{\mu^v esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)}{\lceil(v)\,4D^3}\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\left(\mu-\frac{2D}{\sigma^2}\right)^z(v-z)!}$$

$$(DT-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^i$$

Let $G_{14}(T)=\int_0^\infty U(M)G_6(M,T)dM$, *applying equation* (7)

$$G_{14}(T)=2Db_1\,\mu^v esp\frac{\left[\left(\frac{\sigma^2 b_2 T+2D^2 b_2 T}{2D^2}\right)+\left(\mu+\frac{b_2}{D}\right)\left(\left(\frac{\sigma^2 T}{2}\right)\left(\mu+\frac{b_2}{D}\right)-DT\right)\right]}{b_2\lceil v\left(\sigma^2 b_2^2+2D^2 b_2\right)}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\left(\mu+\frac{b_2}{D}\right)^2(v-z)!}\binom{v-1-i}{i}(DT-\mu\sigma^2 T)^{v-z-2i}\left(\frac{\sigma^2 T}{2}\right)^1+\frac{b_1}{b_2}\frac{\mu^v}{\lceil v}$$

$$esp\left(\frac{\mu^2\sigma^2}{2}-DT\mu\right)\left[\sum_{i=0}^{\frac{v}{2}}\binom{v-z}{i}(DT-\mu\sigma^2 T)^{v-2i}\left(\frac{\sigma^2 T}{2}\right)^i-DT\sum_{i=0}^{\frac{v-1}{2}}\binom{v-1-i}{i}\right.$$

$$(DT-\mu\sigma^2 T)^{v-1-2i}\left(\frac{\sigma^2 T}{2}\right)^i-\frac{\sigma^2 b_1\mu^v}{D(\sigma^2 b_2+2D^2)\lceil v}\,esp\left(\frac{\mu^2\sigma^2 T}{2}-D\mu T\right)$$

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$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\left(\mu+\frac{b_2}{D}\right)^{z}(v-z)!}\binom{v-z-i}{i}\,(DT-\mu\sigma^2T)^{v-z-2i}\left(\frac{\sigma^2T}{2}\right)^{1}-\frac{b_1}{b_2}\frac{\mu^v}{\lceil v}$$

$$\sum_{z=1}^{v}\sum_{i=0}^{\frac{v-2}{2}}\frac{(v-1)!}{\mu^z(v-z)!}\binom{v-z-i}{i}(DT-\mu\sigma^2T)^{v-z-2i}\left(\frac{\sigma^2T}{2}\right)^{1}$$

Hence the inventory costs for (M, T) when the supply is random and lead is constant is

$$C=\left(\frac{Rc+S}{T}\right)+\frac{hcv}{\mu}-\;hc\;\left(\frac{DL+DT}{2}\right)+hc\;G_{11}(T+L)-\big(G_{11}(\mathrm{L})\big)+\frac{1}{\mathrm{T}}\big(G_{14}(T+L)-G_{14}(L)\big)$$

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